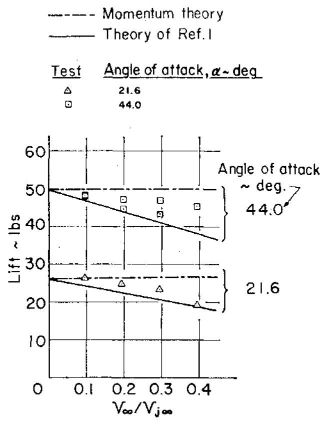


**Fig. 4 Comparison between calculated and measured lift at approximately constant mass flow (calculation based upon measured $\Delta H$ and mass flow).**

equality of pressure on either side of the interface are satisfied without approximations, assuming no roll up. By a conventional momentum-flux control-surface method the normal and axial force components are in Ref. 1 shown to be, respectively,

$$Z = \rho A_{j\infty}(1 - \mu)u_\infty w_\infty \tag{1}$$

$$X = -\rho A_{j\infty}\left\{u_j(u_j - u_\infty) + \frac{1-\mu}{\mu^2}w_\infty{}^2 \times \left[(1+\mu^2)\left(1 - \frac{1-\mu}{4\mu}\ln\frac{1+\mu}{1-\mu}\right) - \frac{1-\mu}{2}\right]\right\} \tag{2}$$

The velocity components $u_\infty$, $w_\infty$, and $u_j$ are defined in Fig. 5. Resolving the resultant force into components normal to and along the freestream direction, the lift and drag of a slanted jet engine in a stream become

$$L = \dot m_j(u_j - V_\infty \cos\alpha)\sin\alpha + [2(1-\mu)\sin\alpha - 2(C-\mu)\sin^3\alpha]q_\infty A_{j\infty} \tag{3}$$

$$D = -\dot m_j(u_j - V_\infty\cos\alpha)\cos\alpha + [2(C-\mu)\sin^2\alpha\cos\alpha]q_\infty A_{j\infty} \tag{4}$$

where $q_\infty$ is the freestream dynamic pressure, and

$$\mu = (V_\infty/V_{j\infty})\cos\alpha/[1 - (V_\infty/V_{j\infty})^2\sin^2\alpha]^{1/2}$$

$$u_j = [2(\Delta H + q_\infty\cos^2\alpha)/\rho]^{1/2}$$

$$A_{j\infty} = \dot m_j/\rho u_j$$

$$C = [1 + (1/\mu^2)]\{\mu - \tfrac{1}{2} + [(1-\mu)^2/4\mu] \times \ln[(1+\mu)/(1-\mu)]\}$$

In dealing with wind-tunnel models having their own external air supply for the jet, a term $\dot m_j V_\infty$ (the inlet drag) must be subtracted from Eq. (4). In writing Eqs. (3) and (4), the tacit assumption has been made that the angle of inclination of the jet in the Trefftz plane is the same as the jet exit angle $\alpha$. This should be valid when $V_\infty \ll V_{j\infty}$.

Equation (3) is plotted on Fig. 4 for comparison with the measured data. It is apparent that the theory of Ref. 1 exhibits the correct trend of lift decrease with increasing forward speed at constant mass flow and angle of attack. Also shown in Fig. 4 for comparison are the constant values of jet lift ($L = \dot m_j V_{j\infty}\sin\alpha$) calculated by momentum theory,

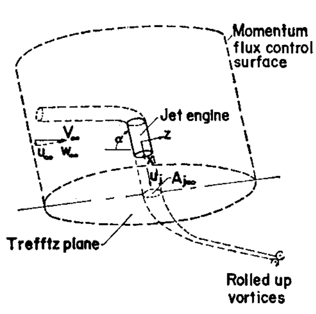


**Fig. 5 Flow model and coordinate system.**

which, of course, does not take into account the trailing vortex system on the interface. It is concluded that the new theory may be of value for improved estimates of the variation of the lift of a slanted round jet with forward speed.

**Reference**


[1] Levinsky, E. S. et al., "Lifting-Surface Theory for V/STOL Aircraft in Transition and Cruise. I," *Journal of Aircraft*, Vol. 6, No. 6, Nov.-Dec. 1969.


# An Explicit Formula for Additive Drag of a Supersonic Conical Inlet


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**Nomenclature**

$M$ = Mach number
$P$ = static pressure
$P_t$ = total pressure
$\infty$ = subscript to $M$, $P$ or $P_t$ indicating parameter is evaluated for freestream conditions, otherwise parameter is evaluated on ray (or conical surface) from cone tip to inlet lip
$\gamma$ = ratio of specific heats
$\theta$ = half angle of conical surface from cone tip to lip
$\phi$ = direction of flow through conical surface relative to inlet axis

CHARTS of mass-flow ratio and additive-drag coefficient for supersonic inlets with a conical centerbody are presented by Mascitti[1] and Barry.[2] Mascitti used a definition of additive drag in terms of an integral of gage pressure from the freestream to the inlet lip and, therefore, computed the additive drag by numerical integration. This technique has the disadvantage of requiring a knowledge of the conical flowfield between the conical shock and the inlet lip. However, if an equivalent definition of additive drag as a difference in momentum is used, an explicit equation for the drag, which requires a knowledge of the flow only in the freestream and through the conical surface intersecting the inlet lip with apex at the tip of the conical centerbody, may be derived. This equation was used by UAC[2] in 1958 and was known at least six years earlier.

The formula for additive drag coefficient is

$$C_{D,a} = \frac{2}{\gamma M_\infty{}^2}\left[\frac{P}{P_\infty}\left\{1 + \gamma M^2\cos\phi \times \left(\frac{\sin(\theta-\phi)}{\sin\theta}\right)\right\} - 1\right] - 2\frac{w}{w_\infty}$$

where the mass-flow ratio is

$$\frac{w}{w_\infty} = \frac{\sin(\theta-\phi)}{\sin\theta}\frac{P_t}{P_{t\infty}}\frac{M}{M_\infty}\left(\frac{1 + (\gamma-1)M_\infty{}^2/2}{1 + (\gamma-1)M^2/2}\right)^{\frac{(\gamma+1)}{2(\gamma-1)}}$$

The parameters of flow angle $\phi$, static pressure $P$, total pressure $P_t$, and Mach number $M$ on the ray from the cone tip to the inlet lip in these two equations may be obtained for any ray (lip) angle $\theta$ from published tables of supersonic conical flow, such as Ref. 3.

An approximate solution for $C_{D,a}$ and $w/w_\infty$ is derived in Ref. 4 in order to avoid the numerical integration used in Ref. 1. However, if one uses tables of conical flow[3] to find the shock-wave angle required by the approximate solution,[4] one


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can obtain an exact solution from the equations of this Note as easily by using the same tables. Therefore, these exact equations are recommended.


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[1] Mascitti, V. R., "Charts of Additive Drag Coefficient and Mass-Flow Ratio For Inlets Utilizing Right Circular Cones at Zero Angle of Attack," TN D-3434, 1966, NASA.

[2] Barry, F. W., "Conical Flow Properties for Use in the Design of Supersonic Inlets," Rept. M-1266-1, 1958, UAC Research Dept., East Hartford, Conn.

[3] Sims, J. L., "Tables for Supersonic Flow Around Right Circular Cones at Zero Angle of Attack," SP-3004, 1964, NASA.

[4] Mascitti, V. R., "An Approximate Solution of Additive-Drag Coefficient and Mass-Flow Ratio For Inlets Utilizing Right Circular Cones at Zero Angle of Attack," TN-5537, 1969, NASA.


# Wave Structure of Exhaust from Transonic Aircraft


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IN the discussion of jets exhausting into a moving airstream Pindzola,[1] Ehlers and Strand,[2] and Kawamura[3] use a reflection coefficient $k$ which assumes a value of unity for no reflection. Waldman and Probstein[4] and Hayes and Probstein[5] use a reflection coefficient $R$ which has a value of zero for no reflection. Using $R$ to interpret inviscid jets, one can draw a map in the $M_j - M_\infty$ plane which is of value for understanding aircraft exhausts.

Figure 1 shows the geometry of the reflection at a shear layer. The two reflection coefficients are defined as

$$R = (p_5 - p_4)/(p_4 - p_3) \tag{1}$$

and

$$k = (p_{14} + p_{24} - 2)/(p_{14} - p_{24}) \tag{2}$$

Here $p_{14}$ is ratio of $p_1$ to $p_4$; this is standard notation. In terms of the flow properties the reflection coefficients are given by

$$k = Y_j/Y_\infty \tag{3}$$

and

$$R = (Y_\infty - Y_j)/(Y_\infty + Y_j) \tag{4}$$

where the dimensionless quantity $Y$ is

$$Y = \gamma M^2/(M^2 - 1)^{1/2} \tag{5}$$

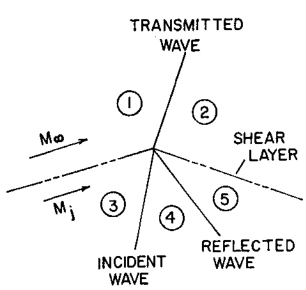


**Fig. 1 Geometry of reflection at a shear layer.**

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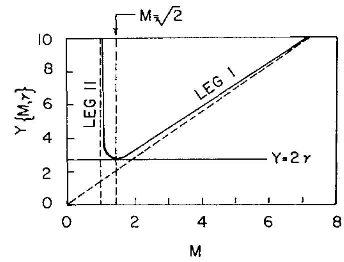


**Fig. 2 Curve of $Y$ vs $M$ for $\gamma = 1.4$.**

Figure 2 shows the behavior of $Y$, and it is this behavior that makes for varied jet phenomena in the transonic region. As can be seen from Eq. (4), it is the relative size of $Y_\infty$ and $Y_j$ which determines the algebraic sign of the reflection coefficient $R$. If $R$ is negative then the reflected wave is the opposite of the incident wave, i.e., an incident compression wave is reflected as an expansion wave.

We are all familiar with the train of shock diamonds that occur from an underexpanded jet. Pictures of static test firing of rockets will show a half dozen or more. Also pictures of exhaust from afterburning turbojets† show similar patterns. In order to have the periodic jet structure it is necessary to have a negative reflection coefficient. The jet must expand, compress, expand, compress, etc. When $R$ is positive for an underexpanded jet there is expansion by fans originating at nozzle exit lip, recompression by imbedded jet shock and then expansion to atmospheric pressure. There is no repetition of the wave pattern.

For a periodic jet, the streamline dividing the jet from the ambient air stream takes on a shape like a corrugated cylinder. For a nonperiodic jet the dividing streamline simply diverges if the jet shock is strong or converges and then diverges once if jet shock is weak. Of course, viscous effects tend to wash out the wave pattern. Even so there remains a different character between the two jets formed with $R$ positive and $R$ negative.

Figure 3 shows the map, which was mentioned earlier, of the different regions for the reflection coefficient. This map is obtained from Eq. (4) along with use of Fig. 2. In the region bounded by $(2)^{1/2} < M_j < \infty$ and $(2)^{1/2} < M_\infty < \infty$ both $Y_j$ and $Y_\infty$ are on leg I of Fig. 2. In the region bounded by $1 < M_j < (2)^{1/2}$ and $1 < M_\infty < (2)^{1/2}$ both $Y_j$ and $Y_\infty$ are on leg II. In other regions $Y_j$ and $Y_\infty$ are on different legs. The curve in Fig. 3 labeled curve A is obtained from the solution of $Y_j = Y_\infty$.

For illustration purposes consider a fixed $M_\infty$ which is represented by dashed line in Fig. 3. Now consider the change in jet character as engine pressure ratio changes resulting in changed $M_j$. From 1 to 2 the jet is periodic; from 2 to 3 it is nonperiodic and once again from point 3 toward higher $M_j$ the jet is periodic. Over a fairly narrow Mach number range of either $M_\infty$ or $M_j$ the jet undergoes pronounced change in wave structure.

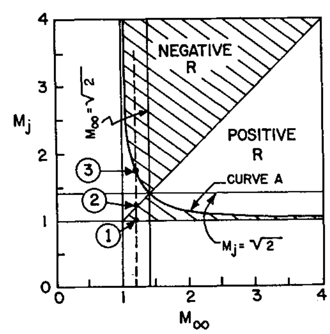


**Fig. 3 Map of reflection coefficient in $M_\infty - M_j$ plane.**

† Due to volume and weight constraints both rockets and turbojets tend to operate underexpanded.